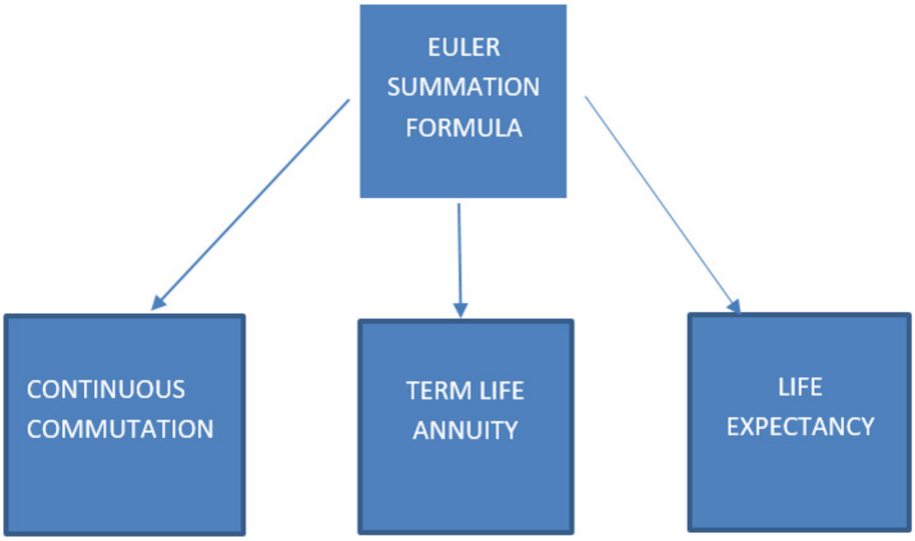
RESEARCH ARTICLE

A theoretical summation-integral scheme involving commutation function in life insurance business

M.G. Ogungbenle*



Highlights

- Applying Euler-Maclaurin formula to estimate life insurance functions used in product pricing and valuations.
- Obtained the value of a continuous commutation function relating the value of a discrete sum to its integral.
- Estimated the n-term life annuity due based on the estimated force of mortality and force of interest.
- Estimated the temporary life expectancy based on the estimated force of mortality and force of interest.

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A theoretical summation-integral scheme involving commutation function in life insurance business

M.G. Ogungbenle*

Department of Actuarial Science, Faculty of Management Sciences, University of Jos, Plateau State, Nigeria.

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Abstract: The aim of this paper is to analytically extend Euler's summation-integral quadrature to core actuarial functions based on sound judgement of numerical analytics. Specifically, the objectives are to theoretically (i) Obtain the value of a continuous commutation function relating the value of a discrete sum to its integral (ii) Estimate the n-term life annuity due based on the estimated force of mortality and force of interest using alternative mathematical technique (iii) Estimate the temporary life expectancy based on the estimated force of mortality and force of interest. One of the most relevant applications of this paper is to provide a sound estimate of commutation functional values used in life and pension funds valuation. We used the elementary summation-integral formula of Euler-Maclaurin to relate the value of a discrete sum $\sum_{t=n}^{k} C_{x+t}$ to its integral $\int_{t=n}^{\omega-x} C_{x+t}dt$ in terms of the derivatives of a continuous commutation function $C(x,t)$ at two distinct points a and b specified in the integral and a remainder term $R(x,t)$. We assume the remainder term $R(x,t) \to o(1)$ where $o(1)$ is small tending to zero as b grows very large and this can be used to compute asymptotic expansions for sums.

Keywords: Asymptotic; annuity; continuous; indicator; Euler-Maclaurin.

INTRODUCTION

When underwriting life assurance policies, the underwriter is aware that the policyholder's obligations are often dependent on a series of actuarial variables that significantly impact on the insured's survival function. These obligations are the summary of live assurance fund's promised benefit to the insured based on their current level of accumulated wealth under minimum guaranteed return. These variables are computed by Actuaries to obtain the insurance premiums quota. In a bid to charging the appropriate premiums for the insured, underwriters apply the experience ratings that are dependent on information content of the past and the future.

Actuaries collect life assurance data on claims from policies, number of policies and sum assured at varying ages. Following Ogungbenle and Ogungbenle (2020), Ogungbenle and Adeyele (2020a), Ogungbenle and Adeyele (2020b), and Ogungbenle and Adeyele (2018c), the data collected from various types of insurance products such as annuities, whole life and endowment assurance are then actuarially combined to prepare life table used in insurance and annuity computations through commutation functions. The correct estimation of age specific death probabilities remains a core objective of life office because it impacts on the valuation of life insurance products. Since the profits and social security of insured depends on survival function, the commutation function used in the mortality table have economic implications. The life office who manages these mortality tables try to establish actuarial models for assessing the risk of life insurance products to come up with proactive measures. One of the core issues in actuarial valuation is the determination of the true values of annuity payout especially how to define its values and what numerical analytics should be applied. The value of life and pensions annuity is a crucial actuarial exercise required for both pension trustees such as pension fund administrators and life offices at large since it often describes the major part of the balance sheet account. A relatively small change in the value of annuity may cause a disproportionate change in profit and loss accounts and solvency of life office. Many methods which are characterized with practical useful results are applied in theory of deterministic valuation of life and pension schemes.

In view of Ogungbenle and Adeyele (2020b) and Ogungbenle and Ogungbenle (2020), the mortality a cohort will experience in the future may not be readily known, consequently, core issues evolving in mortality dynamics could be resolved by formulating reasonable assumptions on future mortality trends in order to obtain numerical results with the help of tables of commutation functions based on that mortality. Following Neil (1979), a consideration of what may occur in the future usually depends on the mortality knowledge already experienced by a defined cohort of lives in the past. Consequently, life insurance models constructed all have common denominator of mortality risk but the risk change in different settings. In actuarial literature, there exist numerical actuarial models of mortality which adopt continuous specialized probability distributions such as Makeham's law of mortality which is appropriate for modeling mortality data. Although these distributions may not necessarily capture all mortality inputs simultaneously such as age, diseases and accidents in mortality analysis. In practice, there are two basic

**Corresponding Author's Email:* moyosiolorun@gmail.com
https://orcid.org/0000-0001-5700-7738

approaches for evaluating mortality which attempt to describe and assess the actual mortality trends. (i) Under the first approach, a life table is constructed while mortality time series follows from its elements. (ii) Under the second approach, the mortality trend is defined by a function characterized by some known probability distributions assumed to be continuous and smooth function of age. Where parametric distribution is involved, the adoption of an actuarial function in the initial estimations of mortality rate or instantaneous mortality intensity is of prime importance to obtain a functional relationship between the parameters. The technique of removing random errors in mortality data (normalization) will then be performed either by modifying a mortality function at every age or by adjusting more functions to mortality data incrementally. However, non-parametric techniques are applicable to life tables by combining mortality data in various values of age x. Life underwriters classify similar risks together depending on actuarial selection rules but the rule may not result in uniform pricing among policy holders. However, the premium will be fair and ensures that the insured pays equitable and competitive premiums depending on the insured risk introduced to the group. The correct criteria in concluding a typical mortality study comprises policy date, issue age, sex, sum assured in force and policy status, termination date, value of claim settled if separate from sum assured value, rating data on substandard risks, cause of death, and past profile of underwriting format. From all these indicators, the core risk selection factors which showing adequate proof to demonstrate the basis of mortality study are gender, age at issue, sum assured value and the choice rating. Both sum-assured value and the choice rating demonstrate a mixture of underwriting conditions which are imposed on the group lives. Since risk factors evolve over time leading to mortality curve having a selection period that subsequently develops in ultimate mortality which in other words is the expected mortality at a defined age when the effect of underwriting would have worn-off, it is therefore necessary to measure policy duration at issue. Consequently, life insurance pricing is obtainable based on the following conditions: The frequency of premium payment; mortality tables; types of life insurance policies varying in terms of the covered risk.

Our objective is to adopt a numerical technique applicable in the approximation of mortality table functions so as to:

(i) Obtain a numerical value of a continuous commutation function relating the value of a discrete sum to its integral (ii) Estimate the n-term life annuity due based on the estimated force of mortality and force of interest through an alternative mathematical technique (iii) Estimate the temporary life expectancy based on the estimated force of mortality and force of interest.

This work describes the main principles of Euler's theory applied in life assurance calculation techniques and their mutual relationships in a way of actuarial descriptions and in a state of mathematical formulation.

The most relevant result is that the method outputs specified formulae for estimating mortality functions employed in preparation of mortality tables. Pricing life assurance policies like pensions remains a major goal of any life office which sells such policies. The policy which produces different premiums and profit margin will tend to be an opportunity for life offices to evaluate the type of premium to charge. Consequently, this study will serve as a good numerical tool in pricing and valuating life insurance schemes with standard formulas especially when single premiums and actuarial present values on different schemes are needed such that the evaluation procedures in the calculation of premiums and actuarial present values are to be minimized.

Mathematical preliminaries of commutation function

Numerical calculation in life contingencies is based on the actuarial present value of some payment made either on the death of the insured person or periodically as long as the insured live continues to survive. In Neil (1997), the primary computation working tool of pension, demography, life contingency mathematics and valuation rests with the correct use of life table that tabulates commutation function such as: l_x and $\overline{a}_x$ at integer ages x where l_x represents the expected number of survivors at age x out of the large cohort l_0 of lives surviving at some age but not necessarily at birth and $\overline{a}_x$ defines the continuous life annuity of a life aged x. These are the inputs of the commutation function.

The influence of mortality which progressively depletes a cohort of insured population is tabulated when the number of insured (Policyholder) survives every integral age out of a group of insured known to be alive at some initial age. From the mortality table, core actuarial functions can be computed. The mortality table can either be probabilistic or deterministic in nature. The probabilistic mortality assumes that the complete future life-time of an insured is a random function and hence there is a probability of death at every time period so that the number of deaths at every integral age is also random. The deterministic mortality assumes exactly how many insured will die at every age. In order that both deterministic and probabilistic mortality framework be associated together, the expected value approach is adopted to functionally bridge them. When the expectation function operates on a random present value of benefit associated with a continuous life insurance model, then we arrive at the actuarial present value function of benefit. In this work, it is intended to develop actuarial nomenclature for differing actuarial present values assuming a deterministic and constant rate of interest i per annum effective over which annuity functions depend. It is instructive to note that we adopt a constant rate of interest for ease of actuarial computation as part of the assumptions. Commutation functions are powerful tool used in life insurance pricing and in valuation especially in evaluating net single premiums and actuarial present values for various insured scheme. Through commutation functions intermediate values are tabulated and the premium values are expressed as functions of these intermediate values. Following Agnes and Jawwad (2011) and Ogungbenle and Adeyele (2018b) all summation goes from $x = y$ to ∞ for the commutation function defined below.

$D_x = v^x l_x = \left(\frac{1}{1+i}\right)^x l_x$ is discounted live, where $v = 1/(1+i)$ is discount term, and l_x is the expected number of survivors at same age x.

$N_x = \sum_{y=x}^{\infty} D_y = \sum_{y=x}^{\infty} v^y l_y$ is the sum of discounted live.

$$S_x = \sum_{y=x}^{\infty} N_y = \sum_{y=x}^{\infty}\sum_{u=y}^{\infty} D_u = \sum_{y=x}^{\infty}\sum_{u=y}^{\infty} l_u v^u = \sum_{m=0}^{\infty}(m+1) l_{x+m} v^{x+m} = \sum_{m=0}^{\infty}(m+1) D_{x+m}$$

This could be applied to ease out computations involving mathematically increasing or decreasing annuities.

$C_x = \left(\frac{1}{1+i}\right)^{x+1}(l_x - l_{x+1}) = v^{x+1}dx = \int_0^1 l_{x+t}\mu_{x+t}dt$ is the discounted death.

Furthermore, it is clear that $C_x = l_x q_x = \int_0^1 l_{x+t}\mu_{x+t}dt$

In Neil (1979), $l_{x+t}\mu_{x+t}$ is referred to as the curve of death and

$$l_x = \int_0^{\infty} l_{x+t}\mu_{x+t}dt = \int_0^{\omega-x} l_{y+t}\mu_{y+t}dt \quad , \quad \mu_{x+t} = -\frac{dl_{x+t}}{l_{x+t}d(x+t)}$$

is the force of mortality intensity $\frac{d^2 l_{x+t}}{dt^2} = 0$ as $\frac{dl_{x+t}}{l_{x+t}} = 0$

The force of mortality is either locally constant or is parametrically defined.

$M_x = \sum_{y=x}^{\infty} C_y = \sum_{y=x}^{\infty} v^{y+1} d_y$ is the sum of discounted death and the functional relationship $d_{(x+m)}/l$ holds

$$R_x = \sum_{y=x}^{\infty} M_y = \sum_{t=0}^{\infty}(t+1)C_{x+t} = \sum_{y=x}^{\infty}\sum_{t=y}^{\infty} C_t = \sum_{y=x}^{\infty}\sum_{t=y}^{\infty} d_t v^{t+1}$$

Life table express values up to a definite maximum terminal age. In Agnes & Jawwad (2011), the actual calculations for the commutation functions therefore would terminate at the smallest age beyond which no life exists in the required life table.

For a complete description of the computational process, we need the intensity of the continuous rate of interest.

Suppose $A(S_1, S_2)$ is the fund about which 1 unit of capital will be invested at time S_1 to accumulate at time S_2 with $S_1 < S_2$ and by the consistence principle $A(S_1,S_2) = A(S_1,\zeta)A(\zeta,\xi)A(\xi,S_2)$ such that $S_1 < \zeta < \xi < S_2$.

The continuous rate of interest intensity is defined by the following differential equation: as follows $\delta(s) = \frac{1}{A(o,s)}\frac{dA(o,s)}{ds} = \frac{d\log_e A(0,s)}{ds}$ under the boundary condition $A(0,0) = 1$.

$A(0,s) = e^{\int_0^s \delta(\zeta)d\zeta}$. If $\delta(s) = \delta$ is constant, then $\delta = \log_e(1+i)$

Summation formula for commutation function

We observe (Berndt, 1975; Borwien and Dilcher; 1989; Lampret; 2001; and Jozef and Bjorn; 2004) investigated mathematical summation technique to deal with commutation function in life assurances. The derivatives below were evaluated with the primary purpose of guiding researchers on how the end results can be applied in actuarial valuations of insured schemes.

$$\frac{1}{D_x}\sum_{t=n}^{k} D_{x+t} = \frac{1}{D_x}\sum_{t=n}^{\omega-x-1} D_{x+t} - \frac{1}{D_x}\sum_{t=k+1}^{\omega-x-1} D_{x+t} \tag{1}$$

where ω is the limit of life

$$\frac{1}{D_x}\sum_{t=1}^{k} D_{x+t} = \frac{1}{D_x}\sum_{t=n}^{\omega-x-1} D_{x+t} - \frac{1}{D_x}\sum_{t=k}^{\omega-x-1} D_{x+t} + \frac{1}{D_x}D_{x+k} \tag{2}$$

$$\frac{1}{D_x}\sum_{t=n}^{k} D_{x+t} = \frac{1}{D_x}\sum_{t=0}^{\omega-x-1} D_{x+t+n} - \frac{1}{D_x}\sum_{t=k}^{\omega-x-1} D_{x+t+k} \tag{3}$$

$$\frac{1}{D_x}\sum_{t=n}^{k} D_{x+t} = \frac{1}{D_x}\int_n^{\omega-x} D_{x+t}dt + \frac{1}{D_x}\frac{D_{x+n}}{2} - \frac{1}{D_x}\int_k^{\omega-x} D_{x+t}dt + \frac{1}{D_x}D_{x+k}$$
$$-\frac{1}{D_x}\frac{D_{x+k}}{2} + \frac{D_{x+k}}{D_x} + \frac{1}{D_x}\sum_{n=2}^{\infty}\frac{B_n}{n!}\frac{d^{n-1}D_{x+n}}{dt^{n-1}} + \frac{1}{D_x}\sum_{n=2}^{\infty}\frac{B_n}{n!}\frac{d^{n-1}D_{x+n}}{dt^{n-1}} \tag{4}$$

$$\frac{1}{D_x}\sum_{t=n}^{k} D_{x+t} = \frac{1}{D_x}\int_n^{\omega-x} D_{x+t}dt + \frac{D_{x+n}}{2D_x} - \frac{1}{D_x}\int_k^{\omega-x} D_{x+t}dt + \frac{D_{x+k}}{D_x}$$
$$+\frac{1}{D_x}\sum_{n=2}^{\infty}\frac{B_n}{n!}\frac{d^{n-1}D_{x+t}}{dt^{n-1}}\Big|t=n + \frac{1}{D_x}\sum_{n=2}^{\infty}\frac{B_n}{n!}\frac{d^{n-1}D_{x+t}}{dt^{n-1}}\Big|t=k \tag{5}$$

$$\sum_{t=n}^{k} A_{x:\frac{1}{t|}} = \int_n^{\omega-x} A_{x:\frac{1}{t|}}dt + 0.5\times A_{x:\frac{1}{n}} - \int_k^{\omega-x} A_{x:\frac{1}{t|}}dt + A_{x:\frac{1}{k|}}$$
$$+\sum_{n=2}^{\infty}\frac{B_n}{n!}\frac{d^{n-1}A_{x:\frac{1}{t|}}}{dt^{n-t}}\Big|t=n + \sum_{n=2}^{\infty}\frac{B_n}{n!}\frac{d^{n-1}A_{x:\frac{1}{t|}}}{dt^{n-1}}\Big|t=k \tag{6}$$

Beginning the summation at $t = 0$.

$$\sum_{t=0}^{n} A_{x:\frac{1}{t|}} = \int_0^{\omega-x} A_{x:\frac{1}{t|}}dt + 0.5\times A_{x:\frac{1}{n|}} - \int_0^{\omega-x} A_{x:\frac{1}{t|}}dt + A_{x:\frac{1}{k|}} +$$
$$\sum_{n=2}^{\infty}\frac{B_n}{n!}\frac{d^{n-1}A_{x:\frac{1}{t|}}}{dt^{n-1}}\Big|t=n + \sum_{n=2}^{\infty}\frac{B_n}{n!}\frac{d^{n-1}A_{x:\frac{1}{t|}}}{dt^{n-1}}\Big|t=k \tag{7}$$

$$\sum_{t=0}^{n} A_{x:\frac{1}{t|}} = \int_0^{\omega-x}\left(\frac{1}{(1+i)}\right)^t P(x,t)dt + 0.5\left(\frac{1}{(1+i)}\right)^n P(x,n)$$
$$-\int_0^{\omega-x}\left(\frac{1}{(1+i)}\right)^t P(x,t)dt + \left(\frac{1}{(1+i)}\right)^k P(x,k) +$$
$$\sum_{n=2}^{\infty}\frac{B_n}{n!}\frac{d^{n-1}\left(\frac{1}{(1+i)}\right)^t P(x,t)}{dt^{n-1}}\Big|t=n + \sum_{n=2}^{\infty}\frac{B_n}{n!}\frac{d^{n-1}A_{x:\frac{1}{t|}}}{dt^{n-1}}\Big|t=k \tag{8}$$

where B_n are Bernoulli numbers and $P(x,t)$ defines the robability function that (x) survives to age $(x+1)$.

In Neil (1977) and Rotar (2007), annuities and life insurance premiums are often paid continuously more frequently than annually such as monthly and weekly although monthly payment is quite common than other frequencies. We can infer from (Berndt, 1975; Jozef and Bjorn, 2004) that annuities payable continuously are most often used as approximation to annuities payable very frequently such as daily or weekly. The expected present values of annuities can then be evaluated numerically but with approximate values of the survival probabilities at integral terms

MATERIALS AND METHODS

In the numerical estimation of mortality function problem, it is necessary for an actuary to establish the governing actuarial equation following functional laws and choose appropriate technique to interpret the result after solution. It seems there has not been a study to compare Neil's estimation methods with Euler's scheme, even the few existing papers show only surface analysis on the use of Euler's in life contigency. This problem has motivated the present study.

In view of Pinelis (2017), the Euler-Maclaurin model associates sum of a commutation obtained at evenly spaced values to the appropriate estimated integral leading to a progressive technique of computing corrections in form of the derivatives of the commutation calculated at the end points

Let $(H, \Omega, \Pr(H))$ be a probability space where H is an event in the sample space Ω with probability density function $\Pr(H)$ where $\Pr(.)$ is a probability function defined on (H, Ω) such that $\Pr(H): H \to [0,1]$ since a random variable can be written in terms of indicator function.

Let $1_H(\psi)$ be a random variable taking values 1 and 0 with probabilities $\Pr(H)$ and $\Pr(H^c)$.

In Rotar (2007), we observe that $1_H(\psi)$ has expectation $\Pr(H)$ and variance $\Pr(H)$ {1- $\Pr(H)$}.

We define 1_H be the indicator function of the event H which takes 1 and 0 such that

$$1_H(\psi): \Omega \to R$$
$$1_H(\psi)\begin{cases} 1 & \psi \in H \\ 0 & \psi \in \Omega - H \end{cases}$$
$$\Omega - H = H^C \tag{9}$$

$H: \Omega \to R$, for real R, then

$$E(H) = \int HdP = \int H(\psi)P(d\psi) = \sum_{\psi\in\Omega} H(\psi)P(\{\psi\}) \tag{9a}$$

If $B \subset \Omega$, such that $E(1_B) = P(B)$, then

$$E(H \times 1_B) = \int_B HdP = \int_B H(\psi)P(d\psi) = \sum_{\psi\in B} H(\psi)P(\{\psi\}) \tag{9b}$$

A discrete random variable X is expressed as a linear combination of indicator random variables $X = \sum_j b_j 1_{H_j}$ for some collection of events $H_j, j \geq 1$ and real numbers $b_j, j \geq 1$. We can obtain the expectation and variance of a random variable X easily by expressing it in this form. Then using the knowledge of the expectation and variance of the indicator variable 1_{H_j}

$$E(1_H) = \Pr(H) \tag{10}$$

Let $\zeta(t)$ be the rate of an annuity payment whose present value is $e^{-\delta t}\zeta(t)$. Then the present value of total of such payment when payments are done weekly, daily, or hourly under continuous framework is $\int_0^{\omega-x} e^{-\delta t}\zeta(t)1_{\{t\leq\Delta\}}dt$.

We note that for $t > \Delta$, $1_{\{t\leq\Delta\}} = o(1)$ where $o(1)$ is a function that tends to zero and hence we are permitted to integrate within the interval $0 < t < \Delta$

$$\bar{a}_x = E\left(\int_0^{\omega-x} e^{-\delta t}\zeta(t)1_{\{t\leq\Delta\}}dt\right) =$$
$$\int_0^{\infty} e^{-\delta t}\zeta(t)E\left(1_{\{t\leq\Delta\}}\right)dt = \int_0^{\omega-x} e^{-\delta t}\zeta(t)\Pr(t\leq\Delta)dt$$

$$\bar{a}_x = \int_0^{\omega-x} e^{-\delta t}\zeta(t)\Pr(t\leq\Delta)dt \tag{11}$$

$$T(x) = \Delta,$$
$$\Pr(t \leq T(x)) = \Pr(x,t) = e^{-\int_x^{x+t}\mu_y dy} = \Pr(x,t) = ({}_tP_x) \tag{12}$$

$\bar{a}_x = \int_0^{\omega-x} e^{-\delta t}\zeta(t)P(x,t)dt$ letting $\zeta(t) \to 1$, we have

$$\bar{a}_x = \int_0^{\omega-x} e^{-\delta t}\Pr(x,t)dt \tag{13}$$

Let $C(x)$ denote a commutation function of interest such that the following condition is satisfied:

$C(x)$ is infinitely many times differentiable and in particular, it is $2K$ times differentiable for an integer k so that Euler's – Maclaurin can be given correction terms up to the (2k – 1)th derivative

Let $y = C(x)$ be a suitable differentiable commutation function $C: R_+ \to R$ such that $\lim_{x\to\infty} C(x) = 0$

Suppose $a \leq y \leq b$ is partition into n equally spaced intervals such that $b = a+nh$ where each interval is of equal length $h = \frac{b-a}{n}$, n is a positive integer representing the number of divisions of the interval $a \leq y \leq b$, then by the trapezoidal quadrature, the Euler's-Maclaurin sum formula describes an asymptotic expression for commutation function.

$$\int_a^a C(y)dy = h\sum_{j=0}^n C(a+jh) - h\times\frac{g(a)+g(b)}{2}, j = 0,1,2,3,..,n \tag{14}$$

$$\int_a^b C(y)dy = h\sum_{j=0}^n C(a+jh) - h\times\frac{g(a)+g(b)}{2}$$
$$+\sum_{i=1}^{k-1}\frac{B_{2i}}{(2i)!}h^{2!}\left[C^{(2i-1)}(a) - C^{(2i-1)}(b)\right] - \frac{B_{2k}}{(2k)!}h^{2k}C, \quad a \leq t \leq b \tag{15}$$

B_k is the kth Bernoulli number and g defines the functional form of the commutation function in question.

$$\int_a^b C(y)dy = hC(a) + hC(a+1h) + hC(a+2h) + hC(a+3h) + ... + hC(a+nh)$$
$$-h\times\frac{g(a)+g(a)}{2}$$
$$+\frac{B_2}{(2)!}h^2\left[C^{(1)}(a) - C^{(1)}(b)\right] + \frac{B_4}{(4)!}h^4\left[C^{(3)}(a) - C^{(3)}(b)\right]$$
$$+\frac{B_6}{(6)!}h^6\left[C^{(5)}(a) - C^{(5)}(b)\right] \tag{16}$$

$$\int_a^b C(y)dy = hC(a) + hC(a+1h) + hC(a+2h) + hC(a+3h) + ... + hC(a+nh)$$
$$-h\times\frac{g(a)+g(a)}{2} + \tag{17}$$
$$\frac{h^2}{12}\left[C^{(1)}(a) - C^{(1)}(b)\right] - \frac{h^4}{720}\left[C^{(3)}(a) - C^{(3)}(b)\right] + \frac{h^6}{30240}\left[C^{(5)}(a) - C^{(5)}(b)\right] + ...$$

Application to life insurance n-term annuity

n-term life annuity

Life insurance products are purchased to address uncertainties associated with survival probabilities of a policy holder (Insured). Such uncertainties could be stochastic in form as a result of the life annuity model or the uncertainties in the parametric variables or even uncertainties in the model governing the observable data. Because solvency regulatory framework requires that a life office must satisfy the capital adequacy to meet up with future obligations in life annuity underwriting, a comparative analysis between random profile of life annuity portfolio asset and random profile of the portfolio liability need be carried out because of the aforementioned uncertainties in the future annuity markets and uncertainties in future mortality trends.

The age at death X and the related mortality function constitute the fundamental function of life contingencies such as force of mortality.

$$\mu(x) = \lim\left[\frac{\Pr\left(x < X \le x+\delta x \middle| X > x\right)}{\delta x}\right] = \frac{f(x)}{s(x)}.$$

The force of mortality describes the instantaneous death rate at age x given that the insured survives to age x

$$P(x,t) = \Pr\left(T(x) > t\right) = \Pr\left(X > x+t \middle| X > x\right) = \frac{s(x+t)}{s(x)}.$$

In terms of the force of mortality $P(x,s) = \exp\left(-\int_x^{x+s}\mu(t)\,dt\right)$

In order to obtain a temporary (term) assurance that generates a payment provided that if death event happens within a limit period of time, we will set, $a = 0, b = n, h = 1$ and the corresponding function $A_{x:\overset{1}{\overline{n|}}}$ is gotten through

$$\int_0^n C(y)dy = C(0)+C(1)+C(2)+C(3)+...+C(n) - \frac{C(0)+C(n)}{2} + \frac{1}{12}\left[C^{(1)}(0) - C^{(1)}(n)\right] - \frac{1}{720}\left[C^{(3)}(0) - C^{(3)}(n)\right] + \frac{1}{30240}\left[C^{(5)}(0) - C^{(5)}(n)\right] + ... \quad (18)$$

$$C(t) = \left(\frac{1}{1+i}\right)^t P(x,t) = e^{-\delta t}p(x,t) \quad (19)$$

$$C^{(k)}(t) = \frac{d^k C(t)}{dt^k} \quad (20)$$

$$C(0) = \frac{l_x}{l_x} = 1, C(1) = e^{-\delta}P(x,1), C(2) = e^{-2\delta}P(x,2), C(3) = e^{-3\delta}P(x,3), ..., C(n) = e^{-n\delta}P(x,n) \quad (21)$$

$P(x,t)$ is the probability that a life aged x survives to age $x+t$.

$$C(0) = v^0 P(x,0) = 1, C(1) = vP(x,1), C(2) = v^2 P(x,2), C(3) = v^3 P(x,3), ..., C(n) = v^n P(x,n), C(\infty) = 0 \quad (22)$$

$$\frac{\partial p(x,t)}{\partial t} = \frac{1}{l_x} \times \frac{dl_{x+t}}{dt} = -\frac{l_{x+t}}{l_x} \times \mu_{x+t} = -P(x,t)\mu_{x+t}.where, P(x,t) = \frac{l_{x+t}}{l_x} \quad (23)$$

$$\frac{\partial \mu_{x+t}}{\partial t} = \left[\mu_{x+t}\right]^2 \quad (24)$$

$$C^{(1)}(t) = -\delta e^{-\delta t}P(x,t) - e^{-\delta t}P(x,t)\mu_{x+t} = -\delta C(t) - P(x,t)\mu_{x+t}e^{-\delta t} \quad (25)$$

$$C^{(1)}(t) + \delta C(t) = -P(x,t)\mu_{x+t}e^{-\delta t} \quad (26)$$

$$C^{(1)}(0) = -\delta e^0 P(x,0) - P(x,0)\mu_{x+0}e^0 = -\delta - \mu_x = -(\delta + \mu_x) \quad (27)$$

$$C^{(1)}(n) = -\delta e^{-\delta n}P(x,n) - P(x,n)\mu_{x+n}e^{-\delta n} \text{ and } C^{(1)}(\infty) = 0 \quad (28)$$

$$C^{(2)}(t) = -\delta\left[-\delta e^{-\delta t}P(x,t) - e^{-\delta t}P(x,t)\mu_{x+t}\right] - \left\{-\delta e^{-\delta t}P(x,t)\mu_{x+t} - e^{-\delta t}P(x,t)\mu^2_{x+t} + \mu^2_{x+t}e^{-\delta t}P(x,t)\right\} \quad (29)$$

$$C^{(2)}(t) = \delta^2 e^{-\delta t}P(x,t) + \delta e^{-\delta t}P(x,t)\mu_{x+t} + \delta e^{-\delta t}P(x,t)\mu_{x+t} + e^{-\delta t}P(x,t)\mu^2_{x+t} - \mu^2_{x+t}e^{-\delta t}(x,t) \quad (30)$$

$$C^{(2)}(t) = \delta^2 e^{-\delta t}P(x,t) + 2\delta e^{-\delta t}P(x,t)\mu_{x+t} \quad (31)$$

$$C^{(2)}(t) - \delta^2 e^{-\delta t}P(x,t) - 2\delta e^{-\delta t}P(x,t)\mu_{x+t} = 0 \quad (32)$$

$$C^{(2)}(0) = \delta^2 + 2\delta\mu_x \quad (33)$$

$$C^{(2)}(n) = \delta^2 e^{-\delta n}P(x,n) + 2\delta e^{-\delta t}P(x,n)\mu_{x+n} and \cdot C^{(2)}(\infty) = 0 \quad (34)$$

$$C^{(3)}(t) = \delta^2\left[-\delta e^{-\delta t}P(x,t) - e^{-\delta t}P(x,t)\mu_{x+t}\right] + 2\delta\left[-\delta e^{-\delta t}P(x,t)\mu_{x+t} - e^{-\delta t}P(x,t)\mu^2_{x+t} + e^{-\delta t}P(x,t)\mu^2_{x+t}\right] \quad (35)$$

$$C^{(3)}(t) = -\delta^3 e^{-\delta t}P(x,t) - \delta^2 e^{-\delta t}P(x,t)\mu_{x+t} \quad (36)$$
$$-2\delta^2 e^{-\delta t}P(x,t)\mu_{x+t} - 2\delta e^{-\delta t}P(x,t)\mu^2_{x+t} + 2\delta e^{-\delta t}P(x,t)\mu^2_{x+t} \quad (37)$$

$$C^{(3)}(t) = -\delta^3 e^{-\delta t}P(x,t) - 3\delta^2 e^{-\delta t}p(x,t)\mu_{x+t} \quad (38)$$

$$C^{(3)}(t) + \delta^3 e^{-\delta t}P(x,t) + 3\delta^2 e^{-\delta t}P(x,t)\mu_{x+t} = 0 \quad (39)$$

$$C^{(3)}(0) = -\delta^3 - 3\delta^2\mu_x \quad (40)$$

$$C^{(3)}(n) = -\delta^3 e^{\delta n}P(x,n) - 3\delta^2 e^{-\delta n}P(x,n)\mu_{x+n}, C^{(3)}(\infty) = 0 \quad (41)$$

$$C^{(4)}(t) = -\delta^3\left[-\delta e^{-\delta tt}P(x,t) - e^{-\delta t}P(x,t)\mu_{x+t}\right] - 3\delta^2\left[-\delta e^{-\delta t}P(x,t)\mu_{x+t} - e^{-\delta t}P(x,t)\mu^2_{x+t} + e^{-\delta t}P(x,t)\mu^2_{x+t}\right] \quad (42)$$

$$C^{(4)}(t) = \delta^4 e^{-\delta t}P(x,t) + 4\delta^3 e^{-\delta t}P(x,t)\mu_{x+t}$$

$$C^{(4)}(t) - \delta^4 e^{-\delta t}P(x,t) - 4\delta^3 e^{-\delta t}P(x,t)\mu_{x+t} = 0 \quad (43)$$

$$C^{(4)}(0) = \delta^4 + 4\delta^3\mu_x \quad (44)$$

$$C^{(4)}(n) = \delta^4 e^{-\delta n}P(x,n) + 4\delta^3 e^{-\delta n}P(x,n)\mu_{x+n}$$
$$C^{(4)}(\infty) = 0 \quad (45)$$

$$C^{(5)}(t) = \delta^4\left[-\delta e^{-\delta t}P(x,t) - e^{-\delta t}P(x,t)\mu_{x+t}\right] + 4\delta^3\left[-\delta e^{-\delta t}P(x,t)\mu_{x+t} - e^{-\delta tt}P(x,t)\mu^2_{x+t} + e^{-\delta tt}P(x,t)\mu^2_{x+t}\right] \quad (46)$$

$$C^{(5)}(t) = -\delta^5 e^{-\delta t}P(x,t) - \delta^4 e^{-\delta t}P(x,t)\mu_{x+t} - 4\delta^4 e^{-\delta t}P(x,t)\mu_{x+t} - 4\delta^3 e^{-\delta t}P(x,t)\mu^2_{x+t} + 4\delta^3 e^{-\delta t}P(x,t)\mu^2_{x+t} \quad (47)$$

$$C^{(5)}(t) = -\delta^5 e^{-\delta t}P(x,t) - 5\delta^4 e^{-\delta t}P(x,t)\mu_{x+t} \quad (48)$$

$$C^{(5)}(t) + \delta^5 e^{-\delta t}P(x,t) + 5\delta^4 e^{-\delta t}P(x,t)\mu_{x+t} = 0 \quad (49)$$

$$C^{(5)}(0) = -\delta^5 - 5\delta^4\mu_{x+t} \quad (50)$$

$$C^{(5)}(n) = -\delta^5 e^{-\delta n}P(x,n) - 5\delta^4 e^{-\delta n}P(x,n)\mu_{x+n} \cdot and, C^{(5)}(\infty) = 0 \quad (51)$$

$$C^{(2k-1)}(t) = (-1)^{2k-1}\left[\delta^{2k-1}e^{-\delta t}P(x,t) + (2k-1)\delta^{2k-2}e^{-\delta t}P(x,t)\mu_{x+t}\right] \quad (52)$$

$$C^{(2k)}(t) = \delta^{2k}e^{-\delta t}P(x,t) + 2k\delta^{2k-1}e^{-\delta t}P(x,t)\mu_{x+t}$$
$$k = 1,2,3... \quad (53)$$

$$\int_0^n C(y)dy = C(0)+C(1)+C(2)+C(3)+...+C(n)-\frac{C(0)+C(n)}{2} + \frac{1}{12}\left[C^{(1)}(0)-C^{(1)}(n)\right]-\frac{1}{720}\left[C^{(3)}(0)-C^{(3)}(n)\right]+\frac{1}{30240}\left[C^{(5)}(0)-C^{(5)}(n)\right] \quad (54)$$

becomes

$$\int_0^n C(t)dt = v^0P(x,0)+v^1P(x,1)+v^2P(x,2)+v^3P(x,3)+...+v^nP(x,n) - \frac{v^0P(x,0)+v^nP(x,n)}{2}+\frac{1}{12}\left[-(\delta+\mu_x)-\left(-\delta e^{-\delta t}P(x,n)-P(x,n)\mu_{x+n}e^{-\delta n}\right)\right]- \frac{1}{720}\left[-\delta^3-3\delta^2\mu_x-\left(-\delta^3e^{-\delta n}P(x,n)-3\delta^2e^{-\delta n}P(x,n)\mu_{x+n}\right)\right]+ \frac{1}{30240}\left[-\delta^5-5\mu_x-\left(-\delta^5e^{-\delta n}P(x,n)-5\delta^4e^{-\delta n}P(x,n)\mu_{x+n}\right)\right]+... \quad (55)$$

Following Ogungbenle & Adeyele (2020b), the force of mortality is defined as

$$\mu_x = \left[\frac{3l_{x+4}+36l_{x+2}+25l_x-16l_{x+3}-48l_{x+1}}{12l_x}\right] \quad (56)$$

$$\mu_{x+n} = \left[\frac{3l_{x+n+4}+36l_{x+2+n}+25l_{x+n}-16l_{x+3+n}-48l_{x+1+n}}{12l_{x+n}}\right] \quad (57)$$

$$\int_0^n C(t)dt = v^0P(x,0)+v^1P(x,1)+v^2P(x,2)+v^3P(x,3)+...+v^nP(x,n)- \frac{v^0P(x,0)+v^nP(x,n)}{2}-\frac{1}{12}\left[(\delta+\mu_x)+\delta e^{-\delta n}P(x,n)+P(x,n)\mu_{x+n}e^{-\delta n}\right]- \frac{1}{720}\left[-\delta^3-3\delta^2\mu_x+\delta^3e^{-\delta n}P(x,n)+3\delta^2e^{-\delta n}P(x,n)\mu_{x+n}\right]+ \frac{1}{30240}\left[-\delta^5-5\mu_x+\delta^5e^{-\delta n}P(x,n)+5\delta^4e^{-\delta n}P(x,n)\mu_{x+n}\right] \quad (58)$$

$$\bar{a}_{x:\overline{n}|} = \ddot{a}_{x:\overline{n}|}-\frac{1+P(x,n)v^n}{2}-\frac{1}{12}(\delta+\mu_x)-\frac{1}{12}\delta e^{-\delta n}P(x,n)-\frac{1}{12}P(x,n)\mu_{x+n}e^{-\delta n}+ \frac{\delta^3}{720}+\frac{3\delta^2\mu_x}{720}-\frac{\delta^3e^{-\delta n}P(x,n)}{720}-\frac{3\delta^2e^{-\delta n}P(x,n)\mu_{x+n}}{720}-\frac{1}{30240}\delta^5-\frac{5}{30240}\mu_x+ \frac{1}{30240}\delta^5e^{-\delta n}P(x,n)+\frac{5}{30240}\delta^4e^{-\delta n}P(x,n)\mu_{x+n} \quad (59)$$

$$A_{x:\overset{1}{\overline{n}|}} = P(x,n)\left(\frac{1}{(1+i)}\right)^n \quad (60)$$

Substituting equation (56) in (59), we have

$$\bar{a}_{x:\overline{n}|} = \frac{\bar{N}_x-\bar{N}_{x+s}}{D_x} = \int_0^n\left(\frac{1}{1+i}\right)^s P(x,s)ds = \ddot{a}_{x:\overline{n}|}-\frac{A_{x:\overset{1}{\overline{n}|}}}{2}-\frac{1}{2}- \frac{1}{12}\left[\delta+\frac{3l_{x+4}+36l_{x+2}+25l_x-16l_{x+3}-48l_{x+1}}{12l_x}\right]-\frac{1}{12}\delta e^{-\delta n}P(x,n)- \frac{1}{12}P(x,n)\left[\frac{3l_{x+n+4}+36l_{x+2+n}+25l_{x+n}-16l_{x+3+n}-48l_{x+1+n}}{12l_{x+n}}\right]e^{-\delta n}+ \frac{\delta^3}{720}+\frac{(3)\delta^2\left(\frac{3l_{x+4}+36l_{x+2}+25l_x-16l_{x+3}-48l_{x+1}}{12l_x}\right)}{720} -\frac{\delta^3e^{-\delta n}P(x,n)}{720}-\frac{3\delta^2e^{-\delta n}P(x,n)\left[\frac{3l_{x+n+4}+36l_{x+2+n}+25l_{x+n}-16l_{x+3+n}-48l_{x+1+n}}{12l_{x+n}}\right]}{720}- -\frac{\delta^3e^{-\delta n}P(x,n)}{720}-\frac{3\delta^2e^{-\delta n}P(x,n)\left[\frac{3l_{x+n+4}+36l_{x+2+n}+25l_{x+n}-16l_{x+3+n}-48l_{x+1+n}}{12l_{x+n}}\right]}{720}- \frac{1}{30240}\delta^5+\frac{\left((-5)\frac{3l_{x+4}+36l_{x+2}+25l_x-16l_{x+3}-48l_{x+1}}{12l_x}\right)}{30240}+ \frac{1}{30240}\delta^5e^{-\delta n}P(x,n)+ \frac{5}{30240}\delta^4e^{-\delta n}P(x,n)\left[\frac{3l_{x+n+4}+36l_{x+2+n}+25l_{x+n}-16l_{x+3+n}-48l_{x+1+n}}{12l_{x+n}}\right] \quad (61)$$

This is the main result of this work

In Neil (1979), the deterministic Woolhouse method of evaluating continuous actuarial functions used multinomial expansion with negative power in evaluating life annuity function and consists of infinitely many terms. However, the terms of those multinomials are also multinomial expansion with positive power each of which is very hard to expand to a higher level of requirement when compared with Euler's. Because of the difficulty level of expansion, the multinomial method with negative power is unnecessarily laborious and this accounts for the reason why Euler's numerical method is adopted here. In (Abramowitz and Stegun, 1972; Andrews, Askey and Roy, 1999; Lampret, 2001), it was observed that that the Euler's Maclaurin formula is a series expansion correcting for the error in approximating the integral of a piecewise linear approximating function.

Though Neil (1979) used the conventional deterministic estimators to evaluate life insurance product function, the Euler's scheme tends to be more analytical and consequently demonstrates higher level of numerical scheme. Neil (1979) model for temporary annuity due is

$$\ddot{a}^{(m)}_{x:\overline{n}|} = \ddot{a}_{x:\overline{n}|}-\frac{m-1}{2m}\left(1-\frac{D_{x+n}}{D_x}\right) \quad (61a)$$

Taking the limit of the Neil's annuity, then we have

$$\lim_{m\to\infty}\ddot{a}^{(m)}_{x:\overline{n}|} = \lim_{m\to\infty}\left[\ddot{a}_{x:\overline{n}|}-\frac{m-1}{2m}\left(1-\frac{D_{x+n}}{D_x}\right)\right] \quad (61b)$$

$$\bar{a}_{x:\overline{n}|} = \ddot{a}_{x:\overline{n}|}-\frac{1}{2}\left(1-\frac{D_{x+n}}{D_x}\right) = \ddot{a}_{x:\overline{n}|}+\frac{v^{x+n}l_{x+n}}{2v^xl_x}-\frac{1}{2} \quad (61c)$$

Following Lampret (2001), when a commutation function is analytic and differentiable in a

domain D, Euler's Scheme is therefore good for such function since we can measure the

analytic relationship between the commutation function at every age.

DISCUSSION

The temporary life annuity due can be defined as follows

$$\ddot{a}_{x:\overline{n}|} = \sum_{t=0}^{n-1}\frac{D_{x+t}}{D_x} = \frac{1}{\left(\frac{1}{1+i}\right)^x l_x}\sum_{t=0}^{n-1}\left(\frac{1}{1+i}\right)^{x+t}l_{x+t} = \sum_{t=0}^{n-1}\frac{\left(\frac{1}{1+i}\right)^t l_{x+t}}{l_x} \quad (62)$$

$$l_x = \int_0^{\omega-x} l_{\theta+t}\mu_{\theta+t}dt$$

We note that if , $\frac{dl_{\theta+t}\mu_{\theta+t}}{dt}>0$ then

$\int_0^1 l_{\theta+s}\mu_{\theta+s}ds = l_\theta q_\theta > l_\theta\mu_\theta$ at the beginning of the mortality table

$$\ddot{a}_{x:\overline{n}|} = \sum_{t=0}^{n-1}P(x,t)\left(\frac{1}{1+i}\right)^t \quad (63)$$

$$\ddot{a}_{x:\overline{n}|} = \sum_{t=0}^{n-1}\left(\frac{1}{1+i}\right)^t P(x,t) = \sum_{t=0}^{n-1}P(x,t)e^{-\delta t} \quad (64)$$

If we take limit as $\delta \to 0$

$$\lim_{\delta\to 0} \ddot{a}_{x:\overline{n}|} = \lim_{\delta\to 0} \sum_{t=0}^{n-1} P(x,t)e^{-\delta t} = \sum_{t=0}^{n-1} \lim_{\delta\to 0} P(x,t)e^{-\delta t}$$

$$= \sum_{t=0}^{n-1} P(x,t) = e_{x:\overline{n}|} \quad (65)$$

$$\lim_{\delta\to 0} \bar{a}_{x:\overline{n}|} = \lim_{\delta\to 0} \int_{t=0}^{n} P(x,t)e^{-\delta t}dt = \int_0^t \lim_{\delta\to 0} P(x,t)e^{-\delta t}dt$$

$$= \int_{t=0}^{n} P(x,t)dt = \mathring{e}_{x:\overline{n}|} \quad (66)$$

$$\lim_{\delta\to 0} \bar{a}_{x:\overline{n}|} = \lim_{\delta\to 0} \ddot{a}_{x:\overline{n}|} + \lim_{\delta\to 0} \left\{ -\frac{1+A_{X:\overline{n}|}}{2} - \frac{1}{12}[\delta + \mu_x] - \right.$$

$$\frac{1}{12}\delta e^{-\delta n}P(x,n) - \frac{1}{12}P(x,n)\mu_{x+n}e^{-\delta n} + \frac{\delta^3}{720} + \frac{3\delta^2\mu_x}{720} - \frac{\delta^3 e^{-\delta n}P(x,n)}{720}$$

$$-\frac{3\delta^2 e^{-\delta n}P(x,n)\mu_{x+n}}{720} - \quad (67)$$

$$\left. \frac{1}{30240}\delta^5 - \frac{5}{30240}\mu_x + \frac{1}{30240}\delta^5 e^{-\delta n}P(x,n) + \frac{5}{30240}\delta^4 e^{-\delta n}P(x,n)\mu_{x+n} \right\}$$

The temporary life expectancy is given as

$$\mathring{e}_{x:\overline{n}|} = e_{x:\overline{n}|} - \frac{1+P(x,n)}{2} - \frac{\mu_x}{12} - \frac{P(x,n)\mu_{x+n}}{12} - \frac{5\mu_x}{30240} \quad (68)$$

Substituting equation (56) in (68)

$$\mathring{e}_{x:\overline{n}|} = e_{x:\overline{n}|} - \frac{1+P(x,n)}{2} - \left[\frac{3l_{x+4} + 36l_{x+2} + 25l_x - 16l_{x+3} - 48l_{x+1}}{12\times 12 l_x}\right] -$$

$$P(x,n)\left[\frac{3l_{x+n+4} + 36l_{x+2+n} + 25l_{x+n} - 16l_{x+3+n} - 48l_{x+1+n}}{12\times 12 l_{x+n}}\right] - \quad (69)$$

$$5\left[\frac{3l_{x+4} + 36l_{x+2} + 25l_x - 16l_{x+3} - 48l_{x+1}}{30240\times 12 l_x}\right]$$

If we define $\mu_x \cong \frac{-1}{2}(\log p_{x-1} \times Px)$ (70)

Then the temporary life expectancy becomes

$$\mathring{e}_{x:\overline{n}|} = e_{x:\overline{n}|} - \frac{1+A_{x:\frac{1}{\overline{n}|}}}{2} + \frac{1}{24}(\log p_{x-1} \times p_x) - \frac{1}{12}P(x,n)\mu_{x+n} - \frac{5}{30240}\mu_x \quad (71)$$

Put (70) in (59)

$$\bar{a}_{x:\overline{n}|} = \ddot{a}_{x:\overline{n}|} - \frac{A_{x:\frac{1}{\overline{n}|}}}{2} - \frac{1}{2} - \frac{1}{12}\left[\delta + \frac{-1}{2}(\log p_{x-1} + \log Px)\right] - \frac{1}{12}\delta e^{-\delta n}P(x,n) -$$

$$\frac{1}{12}P(x,n)\left[\frac{-1}{2}(\log p_{x-1} + \log Px)\right]e^{-\delta n} +$$

$$\frac{\delta^3}{720} + \frac{(3)\delta^2\left(\frac{-1}{2}(\log p_{x-1} + \log Px)\right)}{720}$$

$$-\frac{\delta^3 e^{-\delta n}P(x,n)}{720} - \frac{3\delta^2 e^{-\delta n}P(x,n)\left[\frac{-1}{2}(\log p_{x-1} + \log Px)\right]}{720} -$$

$$\frac{1}{30240}\delta^5 + \frac{\left((-5)\frac{-1}{2}(\log p_{x-1} + \log Px)\right)}{30240} +$$

$$\frac{1}{30240}\delta^5 e^{-\delta n}P(x,n) + \frac{5}{30240}\delta^4 e^{-\delta n}P(x,n)\left[\frac{-1}{2}(\log p_{x-1} + \log Px)\right] \quad (72)$$

Equations (61), and (72) are different forms of $\bar{a}_{x:\overline{n}|}$ with Euler-Maclaurin's scheme which are at higher level of rigour than Neil's model $\bar{a}_{x:\overline{n}|}$ in equation (61c).

CONCLUSION

In this Paper we have shown how to use Euler's-Maclaurin equation to approximate life insurance functions. The state of art evaluation of commutation function under the framework of Euler's scheme seems more elegant and computationally superior because of its specifics compared to other methodologies such as Neil's in insurance product pricing. In practical terms, the actuarial valuation and funding of life and pensions revolves round deterministic technique of commutation function whose application is found in equations (55) to (72). For example, in the valuation of defined benefits scheme, the benefit payable at retirement is obtained by pre-determined symbolic expression of actuarial nomenclature which spells out a multiple of accrual factor, service years and final wages expressed in the trust deeds and rule. It is therefore critical that life insurance pricing should choose the required valuation technique under analytic framework to maintain equilibrium in policy holder's and sponsor's interest. Future research work could be carried out where relevant data is applicable.

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DECLARATION OF CONFLICT OF INTEREST

The author declares no conflict of interest.

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